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THE ESTIMATION OF NATURAL FREQUENCIES AND DAMPING RATIOS OF OFFSHORE STRUCTURES.

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ABSTRACT

This paper focusses on the estimation of natural frequencies and modal damping ratios from measured response spectra, with particular emphasis on the dynamic response of offshore structures to wind and wave excitation. At present, estimates of natural frequencies and damping ratios are computed from the location and half-power bandwidths of resonant peaks in a structure's ambient response power spectrum. While reliable natural frequency estimates are typically obtained in this manner, half-power bandwidth damping estimates are shown to be highly sensitive to the method employed in estimating the response spectrum. The lack of confidence bounds on natural frequency and damping estimates further restricts the utility of the estimates. An alternative method is developed based on a powerful method of spectral estimation known as the Maximum Entropy Method (MEM). The resulting technique yields estimates of natural frequencies and modal damping ratios as well as approximate statistics on the reliability of the estimates. Performance of this new method is explored through extensive Monte Carlo simulation of one and two degree-of-freedom systems. Conventional estimates are also simulated for comparison with the MEM parameter estimator. The use of the MEM parameter estimator is further illustrated with ambient response data from Shell Oil's South Pass 62C platform. The MEM parameter estimates show excellent agreement with natural frequency and damping estimates obtained during recent tests conducted using forced excitation.

INTRODUCTION

To date, substantial effort has been expended in attempts to estimate dynamic response parameters of offshore structures. Natural frequencies, mode shapes and damping ratios have been estimated with varying degrees of success. A portion of the effort has been motivated by active research and development of structural integrity monitoring systems. The viability of such techniques is particularly dependent on the ability to accurately estimate natural frequencies and mode shapes. For design purposes, however, accurate estimates of damping ratios are even more important. The future design of fatigue resistant deepwater platforms, whose fundamental periods lie well within energetic portions of the wave spectrum, depend upon accurate knowledge of the damping ratios of existing structures.

At present the most widely practiced method of estimating natural frequencies and damping ratios of an offshore structure utilizes measurements of the platform's response to wind and wave excitation. Time-synchronus acceleration records are typically gathered at different locations on the platform and processed into auto- and cross-spectra using Fast Fourier Transform (FFT) spectral estimators. This convenient representation of the data permits peaks in the spectrum to be associated with "global" modes of vibration. Each natural frequency and damping ratio is typically estimated from the frequency at which a peak occurs and its half-power bandwidth.

This estimation scheme (hereafter referred to as the conventional method) has, for the most part, proved to be a satisfactory means of ascertaining natural frequencies. For this reason the primary emphasis of this paper is the estimation of damping from ambient response spectra.

Conventional Damping Estimation

Half-power bandwiths obtained from a structure's ambient response spectrum have frequently been represented as an absolute measure of the structure's damping. Unfortunately, this ignores the fact that the damping estimates are computed by manipulating samples taken from a realization of a random process (i.e., the acceleration response). Such damping estimates are only realizations of a random variable known as the damping estimator. Accordingly a damping estimate computed from measured data can take on any value allowed by the estimator's probability distribution and thus has little meaning without further information. To demonstrate this point, the conventional damping estimator was applied to the numerically simulated response of a single degree-of-freedom system excitated by Gaussian white noise. A 21 minute time history was created for a resonator with a 1 Hz natural frequency, 3% damping and 30 db

References and illustrations at end of paper.

signal to noise ratio (defined as the ratio of the height of the spectral peak to the noise floor). The record was sampled at 8 Hz and two auto-spectra were computed by the Blackman-Tukey method [1,2]. The first of these spectra was computed with an approximate resolution of 0.16 Hz and is shown in Figure 1 as a solid line. The conventional half-power bandwith damping estimate for this spectrum is 5.3%. In this particular example one might have anticipated an inflated damping estimate on the basis of inadequate resolution. Increasing the resolution to 0.016 Hz produced the spectrum shown as a dashed line in Figure 1. In this case, the damping was estimated at 1.6%. This simple example rather dramatically demonstrates the sensitivity of the damping estimate to one's spectral analysis procedure. Furthermore, there is no guide to indicate the most accurate procedure. The conclusion here is simply that more information about the statistical nature of the estimator is needed in order for an estimate to be useful.

The most frequently used quantity for expressing the statistical nature of an estimator is a confidence interval. Briefly stated, the confidence interval for an estimate defines a range of parameter values surrounding the estimate which has a high probability (typically 95% or 99%) of containing the expected value of the estimator. When the expected value of the damping estimator equals the true damping, the estimator is said to be unbiased and the confidence interval reflects the uncertainty in estimating the true value. For example, if a damping estimate, $\hat{\xi}$ is obtained from an unbiased estimator with a Gaussian probability distribution and variance, σ^2, then one can state with 95% confidence that the true damping is contained in the interval $[\hat{\xi}-1.96\sigma, \hat{\xi}+1.96\sigma]$. Thus the confidence interval provides a measure of how far the estimate may be expected to be from the true value.

In order to implement confidence intervals with damping estimates, one must be able to evaluate the expected value and variance of the estimator. Unfortunately, these statistics are not available within the current state-of-the-art. To gain insight into the magnitude of these statistics, simulation studies were conducted on estimates made from the response of simple one and two degree-of-freedom systems excited by white noise. Briefly, fifty independent response time-histories were numerically generated for systems with known natural frequencies, damping ratios and signal-to-noise ratios. Blackman-Tukey auto-spectra were computed for each realization at thirteen different resolutions and the conventional damping estimator was applied to each of the spectra. The desired ensemble statistics were approximated by the average and variance of the fifty damping estimates obtained for a given resolution. The results for a one degree-of-freedom system with a 1 Hz natural frequency, 3% damping and 30 dB signal-to-noise ratio are shown in Figure 2. In this example each realization consisted of a 21 minute displacement response record sampled at 8 Hz. The triangles shown in the figures represent the value of the average damping or its standard deviation obtained from the fifty realizations. The abscissa in both figures is a nondimensional quantity which can be interpreted in terms of spectral resolution or variance. Since the record length is fixed for the results shown in Figure 2, the lag to record length ratio ($R = L/T$) is inversely proportional to resolution (resolution = $2/TR$) and directly proportional to the variance of the spectral estimator. The dashed line in the top figure shows the true value of the damping and in the bottom figure represents the Cramér-Rao bound on the standard deviation of the estimator. This bound is the smallest standard deviation that any unbiased estimator of the damping can have, for the specified record length and resonator characteristics. Any unbiased estimator which achieves this bound is called an efficient estimator and can be shown to possess numerous desirable properties [3].

The results of this simulation study reveal that the bias and variance of the conventional damping estimator is strongly related to the properties of the spectral estimator. Figure 2 shows the nature of the trade-off between bias and variance that must be made in selecting the resolution.

The results of this study also show that the conventional damping estimator is not efficient. This is demonstrated in Figure 2 by a standard deviation which is substantially larger than the Cramér-Rao bound at lag to record length ratios corresponding to an unbiased estimator. The use of such a non-efficient damping estimator may severly limit the utility of the estimated damping ratio. For example, in Figure 2 a lag to record length ratio of 5% yields an unbiased estimator with a standard deviation 0.75. Putting this information in the form of a confidence interval for an estimate $\hat{\xi}$, one can state with 95% confidence (assuming a Gaussian distribution) that the true damping ratio lies in the interval $(\hat{\xi}-1.5, \hat{\xi}+1.5)$. However, if the estimator had been efficient, the associated confidence interval would be $(\hat{\xi}-0.5, \hat{\xi}+0.5)$. These results indicate that the conventional damping estimator would require substantially longer records to attain confidence intervals equal to those of an efficient estimator. It is important to remind ourselves at this point that the discussion of efficiency and confidence intervals for the conventional damping estimator is completely academic since the estimator statistics cannot be evaluated when working with field data.

It might be argued that an alternative to computing these statistics would be to use very long records and thus ensure small uncertainties in the damping estimates. However, it can be shown that in most cases the record lengths required to achieve this goal are by far too long to be practical. For example, the Cramér-Rao bound shows that for the system associated with Figure 2, an efficient estimator would require 4 hour response records in order to have 5% uncertainty in damping estimates. Since the conventional damping estimator is inefficient one would actually require records considerably longer than 4 hours.

An Alternative to the Conventional Method

The preceding discussion has clearly demonstrated the inability of the conventional damping estimator to provide accurate damping estimates from ambient response time-histories of practical length. In an attempt to overcome the inadequacies of the conventional method, an alternative technique of spectral estimation known as the Maximum Entropy Method was employed. The result was the development of a new natural frequency and damping estimator which capitalizes on MEM's ambient ability to provide smooth, highly resolved response spectra from short time histories and also furnishes approximations for the bias and variance of its estimates.

Prior to the discussion of the new estimators, a brief introduction to MEM will be given in view of its relative newness to the field of vibration testing. The interested reader is refered to reference 4 and its bibliography for more complete details on MEM.

The Maximum Entropy Method of Spectral Estimation

To introduce the philosophy behind the Maximum Entropy Method, it is convenient to describe MEM in terms of its similarity with the Blackman-Tukey spectral estimator. It is recalled that the Blackman-Tukey method computes spectral estimates as the Fourier Transform of autocorrelation functions estimated from finite duration time-histories. Spectra computed with the Blackman-Tukey method can be shown to be inherently limited in resolution as a result of the required truncation of the estimated autocorrelation function and the implicit extension of the truncated function to infinite lag with zeros. The Maximum Entropy Method, on the other hand, seeks to improve resolution by analytically extending the autocorrelation function from its truncation point to infinite lag in a more realistic manner. It is this extension of the autocorrelation function which forms the basis of the maximum entropy spectral estimate.

The extension of the autocorrelation function which MEM achieves can be viewed as the result of fitting a special model to a finite portion of the known autocorrelation function. This model provides the analytic means for extrapolating from "p + 1" samples of a known autocorrelation function, $R(k)$, $k=0, 1, \ldots p$, to obtain the remaining values, $R(k)$, $k=p+1, \ldots \infty$. In a superb mathematical development, which is beyond the scope of this paper, Burg [5] demonstrated that the maximum entropy extension of the autocorrelation function is given by the following pth order recursion equation:

$$R(k) = -A_1 R(k-1) - A_2 R(k-2) - \ldots - A_p R(k-p) \; ; \; k>0 \; \ldots\ldots \quad (1)$$

The parameters $\{A\}$ are obtained as the solution of equation (2) which is commonly known as the Yule-Walker equations. Thus if given p+1 lags of

$$\begin{bmatrix} R(0) & R(1) & . & . & R(p-1) \\ R(1) & R(0) & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ R(p-1) & . & . & R(1) & R(0) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ . \\ . \\ A_p \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \\ . \\ . \\ R(p) \end{bmatrix} \quad (2)$$

an autocorrelation function, one can uniquely define the pth order MEM coefficients $A_1, \ldots, A_p$ and subsequently generate the extended autocorrelation function. To illustrate this concept acceleration data collected on Shell Oil's South Pass 62C [6] and sampled at 6.4 Hz was used to estimate the 50 second (320 lag) autocorrelation function shown at the top of Figure 3. The first 10 seconds of the autocorrelation function were then used to compute the 64th order MEM coefficients which, in turn, were used to extrapolate the next 40 seconds of lag. The resulting function is shown on the bottom of Figure 3. As required MEM duplicated the "known" portion of the autocorrelation function exactly and then extended the function into the unknown region. It is this extension of the autocorrelation function that provides improved resolution in the final spectral estimate.

The infinite duration autocorrelation function determined by equations (1) and (2) provides an appealing conceptual framework for MEM. In practice, however, one need never actually compute the extended function in the course of determining the power spectrum. Instead, it can be shown that the pth order MEM spectrum is uniquely defined as

$$S(f) = \frac{\sigma_p^2 \, \Delta}{\left| 1 + \sum_{k=1}^{p} A_k e^{-j2\pi k f \Delta} \right|^2} \quad \ldots\ldots \quad (3)$$

where: A_k are the pth order MEM coefficients

$$\sigma_p^2 = R(0) - \sum_{k=1}^{p} A_k R(k)$$

Δ is the sampling increment

$j = \sqrt{-1}$

A major impediment to the wide-spread usage of this spectral estimator has been the difficulty selecting the order "p" which produces the optimum estimate. Several criteria have been suggested as an aid in the selection process but as yet no definitive general technique has emerged. In the sections to follow the order selection problem will be defined in terms of the more easily solved task of finding the order which produces the optimum natural frequency or damping estimate.

The MEM Natural Frequency and Damping Estimators

The estimation of natural frequencies and damping ratios from an MEM spectral estimate does not immediately appear to have any great advantage over the conventional method. However when the form of the pth order MEM spectrum is reviewed it is found that the spectral estimate is actually a closed-form equation for the response spectrum. This unique feature of the MEM spectral estimator provides the key to formulating the parameter estimators and their statistics.

The MEM natural frequency and damping estimators are quite easily obtained from the definition of the conventional parameter estimator. For example, the estimation of natural frequencies can be expressed as the selection of frequencies corresponding to relative maxima of the response spectrum. This search for maxima can be recast into a more mathematical form using the result from calculus which states that the derivative of a function is zero at an extrema. Since the spectrum is available in functional form, the relative extrema of the power spectrum can be found by solving

$$\frac{dS(f)}{df} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Thus all natural frequency estimates for a given spectrum can be obtained as solutions of equation (4). Substituting equation (3) into equation (4) and simplifying produces equation (5); the definition of the MEM natural frequency estimator.

$$\sum_{k=0}^{p} \sum_{\ell=0}^{p} kA_k A_\ell \sin[2\Pi(\ell-k)f\Delta] = 0 \quad ..... \quad (5)$$

where: $A_0 = 1$

Similarly, the half-power frequencies are defined in the present context as the pair of frequencies bracketing the natural frequencies which correspond to spectral ordinates 3 dB down from the resonant peak. In more concise terms, the half-power frequencies are solutions of

$$S(f) = \tfrac{1}{2} S(f_n) \quad .... \quad (6)$$

As before, substitution of equation (3) into equation (6) produces the MEM half-power frequency estimator

$$\sum_{k=0}^{p} \sum_{\ell=0}^{p} A_k A_\ell \{\cos[2\pi(k-\ell)f\Delta]$$

$$- 2 \cos[2\pi(k-\ell)f_n\Delta]\} = 0 \quad .... \quad (7)$$

where: $A_0 = 1$

The estimation of natural and half-power frequencies with equations (5) and (7) is easily accomplished using a numerical solution scheme such as Newton's method. These algorithms are easily implemented on a computer and only require an initial estimate of the parameter to solve the estimation equation. The solution of equations (5) and (7) by Newton's method is demonstrated in reference 4.

The MEM estimation equations developed thus far are a convenient means of computing the conventional parameter estimates from MEM spectral estimates. However, it should be recognized that the estimates obtained using equations (5) and (7) are virtually the same as the estimates that would be obtained from the MEM spectrum graphically. The exceptional facet of the analytical formulation is that it provides the means for evaluating the estimator statistics. The expected value and variance of the parameter estimators are found via a rather grueling exercise in mathematics, the details of which may be found in reference 4. It suffices to say in this presentation that the estimator statistics are obtained, after several assumptions, by expanding the estimation equations about the true values of the parameter estimates. The resulting expressions, which are valid to first order, relate the expected value and variance of the estimators to the true value of the estimated quantity and the MEM coefficients. In practice these statistics are evaluated using the estimated parameters since the true values are unknown.

To verify the utility of these first-order approximations to the estimator statistics, the simulation studies conducted on the conventional method were repeated using the MEM estimators. Figure 4 illustrates the typical characteristics of estimates made from the response of a single degree-of-freedom system. The properties of the resonator used in the example are identical to the system described in reference to Figure 2. The format of Figures 2 and 4 are the same with the exception of an additional line added to the bottom of Figure 4. This line is the standard deviation of the estimator predicted by the first-order approximation.

The simulation results for the single degree-of-freedom system reveal that the MEM damping estimator is not an asymptotically efficient estimator due to the presence of a small positive bias (less than 5%) at orders corresponding to attainment of the Cramér-Rao bound. The results reveal that the average damping converges to consistently high values and that the bias decrease with increasing true damping. The most important consequence of this study was the verification of the first order prediction of the estimator statistics. In each test case it was found that the first-order approximation for the standard deviation of the MEM estimators provides an excellent match to the simulation results.

Simulation studies of two degree-of-freedom systems also demonstrated excellent agreement with the theoretically predicted estimator statistics. A typical result from the two degree-of-freedom tests is shown in Figure 5. In this example the system had natural frequencies at 0.5 Hz and 1.0 Hz with 2% damping for both modes. The signal-to-noise ratio was 30 dB for each mode and the record length and sampling frequency were unchanged from the preceding example.

The results presented in Figure 5 show that as the order increases, the average damping estimate exhibits large fluctuations which quickly decay to small cycles about a biased estimate. These studies showed that the MEM damping estimator contains little bias (5% or less depending on the true damping) and a standard deviation very nearly equal to the Cramér-Rao bound when the record length is long and the natural frequencies are well separated. While the exact behavior of the damping estimator is unknown for very small natural frequency separation, it is felt that large systematic biases inherent to half-power damping estimates will dominate the estimation error.

Figure 5 also illustrates the typical agreement between the theoretically predicted and "measured" estimator statistics. Unlike the results presented for the single degree-of-freedom systems, the theoretical standard deviation was found to substantially different from the simulation results for orders corresponding to biased estimates. However, the first-order approximation demonstrated good agreement with the simulation statistics at sufficiently long lags that transients in average dumping had decayed. In all cases where the theory departed from the measured values, the predicted values of the standard deviation were found to exceed the true values. Consequently, many plots of the theoretical standard deviation versus order exhibit a rapid drop followed gentle rise with increasing lag to record length ratio. This trend will prove to be very useful in selecting the optimum parameter estimate when analyzing field data.

PERFORMANCE OF THE MEM DAMPING DAMPING ESTIMATOR WITH FIELD DATA

The MEM damping estimator has thus far been studied in an ideal setting. Theoretical properties of the estimators were developed and characteristics with simulated data were delineated. The information obtained from these analyses are combined in this section into a procedure for the estimation of natural frequencies and dampings from experimentally measured response data. The guidelines for practical parameter estimation are readily illustrated by means of an example analysis.

The data used for this purpose were obtained from ambient response recordings from Shell Oil's South Pass 62C platform. This platform is an eight-leg diagonally braced jacket structure which stands in 327 feet of water. The estimation of this structure's natural frequencies and damping ratios has been the subject of numerous experimental and analytical studies which include a recently conducted full-scale forced vibration test. The platform provides an ideal test for the MEM estimators.

Ambient response measurements were obtained using two accelerometers as described in reference 6. Acceleration response was recorded for 32 minutes using an FM tape recorder. The signal was low-pass filtered at 3 Hz prior to recording to remove high frequency machine noise. Records were digitized with a sampling frequency of 6.4 Hz and autocorrelation functions were computed for each of the records with a maximum lag of 160 seconds. MEM spectral estimates were computed for both end-on and broadside response measurements. The broadside response spectrum is shown in Figure 6. The fundamental broadside and torsion modes are clearly revealed. MEM damping estimates and their corresponding standard deviations were determined for each of the fundamental modes. Estimates were evaluated at 44 different orders ranging from 12 to 100 (i.e., lag = 2.03 - 15.8 seconds). The resulting estimates for the first broadside flexure mode are shown in Figure 7. The top figure shows the parameter estimates obtained at different lag to record length ratios (i.e., orders of the MEM spectra). The standard deviation $\hat{\sigma}$, computed with each of the parameter estimates is plotted in the bottom figure. The standard deviation is also used to compute $\pm\, 2\hat{\sigma}$ confidence bounds which are shown in the top figure as dashed lines. This figure contains sufficient information to select the "best" estimate of the damping for the lowest broadside flexural mode.

The essential task in selecting an optimum estimate is to find the value which the estimates converge to as the lag ratio (i.e., the lag to total record length ratio) increases. The use of this value as the estimate is supported by results of the simulation studies. In each of the cases described in reference 4, the average parameter estimate consistently converged to within a few percent of the true value as the lag ratio increased. Consequently, selection of an estimate from the "region of convergence" provides reasonable assurance of a minimally biased estimate.

Identification of convergence is in most cases a reasonably straightforward process. Examining Figure 7, it is found that the behavior of the parameter estimates as a function of lag ratio is characterized by a period of erratic fluctuations which degenerate to small cyclical variations in the estimates. Similar patterns observed in two degree-of-freedom simulations identify these small variations as characteristic of converging estimates. Simulation results also suggest that additional evidence of convergence is supplied by the presence of a gradual increase in the standard deviation across the region of convergence. While the standard deviations shown for the field data exhibit fairly strong fluctuations, the prescribed trend is clearly visible.

Once convergence of the parameter estimate has been identified, the "best" estimate can be read from the plot or calculated as the numerical average of the estimates in the region of convergence. For example, the region of convergence in Figure 7 can be defined by the lag ratios 0.33% to 0.82%. Accordingly, the best estimate of the damping ratio, determined as the average value over this range of lag ratios is 2.0%. The standard deviation which quantifies the uncertainty in the best estimate is given by the approximate value observed at the onset of convergence. For this example, the standard deviation is approximately 0.3% of critical damping. In view of the judgment required to locate the region of convergence and the approximate nature of the standard deviation calculation, estimates should be limited to two for damping ratios, and one for standard deviations.

This selection procedure was applied to the other fundamental modes. The resulting damping estimates are shown in Table 1 with their corresponding $\pm\, 2\sigma$ uncertainties. Dampings were also computed from Blackman-Tukey spectral estimates using a maximum lag of 160 seconds. These results are included in Table 1 along with estimates obtained by J. A. Ruhl [6] using forced vibration. The MEM damping estimates show very good agreement with Ruhl's transient decay tests. In each case, Ruhl's estimates computed over cycles 0-4 and 0-10 are included in the confidence interval for the MEM estimates. Conversely, damping estimates compute from Blackman-Tukey spectra severely underpredict the damping. However, the use of slightly different resolution (i.e., alternate windows and/or maximum lags) produced damping estimates which significantly overpredicted the damping. In these cases, there was no apparent means of choosing the appropriate estimate.

Similar results were reported by Enochson [7] who analyzed 27.6 minutes of acceleration response data from South Pass 62C using a digital Laplace transform technique. Depending on the resolution selected, his reported damping estimates for the lowest flexural mode varied from 0.3% to 2.6% with no indication of the most reliable result.

CONCLUSIONS

1. Half-power bandwidth damping estimates made from conventionally obtained ambient response spectra were shown to be an unreliable measure of the true damping.

2. Confidence intervals were suggested as a convenient means of including the probabalistic nature of the damping estimation problem.

3. Lacking the information required to construct confidence intervals for conventional damping estimates, simulations studies were used to demonstrate the nature of the bias and variance in the damping estimator. The variance was shown to be too large to neglect under normal circumstances.

4. A new technique for estimating natural frequencies and damping ratios was developed using a powerful method of spectral estimation known as the Maximum Entropy Method. The resulting technique yields parameter estimates as well as the information required to construct approximate confidence intervals.

5. Simulation studies demonstrated the utility of the MEM estimation statistics and exposed a

simple scheme for obtaining the "optimum" parameter estimates.

6. Use of the MEM damping estimator was illustrated with ambient response data from Shell Oil's South Pass 62C platform. The results showed excellent agreement with recent forced vibration tests.

NOMENCLATURE

$A_1, A_2, \ldots, A_p$ = pth order MEM coefficients

$R(k)$ = autocorrelation function at lag=$k\Delta$

$S(f)$ = power spectral density function

f = frequency in Hertz

f_n = natural frequency in Hertz

p = order of MEM spectral estimate

Δ = sampling increment in seconds

ξ = critical damping ratio in percent

σ = standard deviation

σ_p^2 = scale factor for pth order MEM spectrum

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REFERENCES

[1] Blackman, R.B., and Tukey, J.W., The Measurement of Power Spectra, Dover, New York, 1958.

[2] Jenkins, G.M. and Watts, D.G., Spectral Analysis and Its Applications, Holden-Day, San Francisco, CA, 1969.

[3] Cramér, H., Mathematical Methods of Statistics, Princeton University Press, Princeton, NJ, 1946.

[4] Campbell, R.B., "The Estimation of Natural Frequencies and Damping Ratios of Offshore Structures", Ph.D. Thesis, Massachusetts Institute of Technology, 1980.

[5] Burg, J.P., "Maximum Entropy Spectral Analysis", Ph.D. Thesis, Stanford University, 1975.

[6] Ruhl, J.A., and Berdahl, R.M., "Forced Vibration Tests of a Deepwater Platform", Proceedings of the 1979 Offshore Technology Conference, Paper No. 3514, pp. 1314-1354, Houston, May 1979.

[7] Enochson, L., "A Digital Technique for Damping Measurement of Fixed Platforms". Proceedings of the 1976 Offshore Technology Conference, Paper No. 2556, pp. 375-394, Houston, May 1976.

TABLE 1
SOUTH PASS 62C DAMPING RATIO ESTIMATES FOR FIRST-ORDER MODES

			J. A. RUHL – TRANSIENT DECAY DAMPING OVER CYCLES		
	BLACKMAN-TUKEY	MEM	0-4	5-9	0-10
BROADSIDE	1.14	2.0 ± 0.6	1.65	0.86	1.38
END-ON	0.45	2.1 ± 0.6	1.72	1.29	1.53
TORSION	0.27	1.3 ± 0.4	1.20	1.03	1.11

ALL DAMPING RATIOS MEASURED IN %

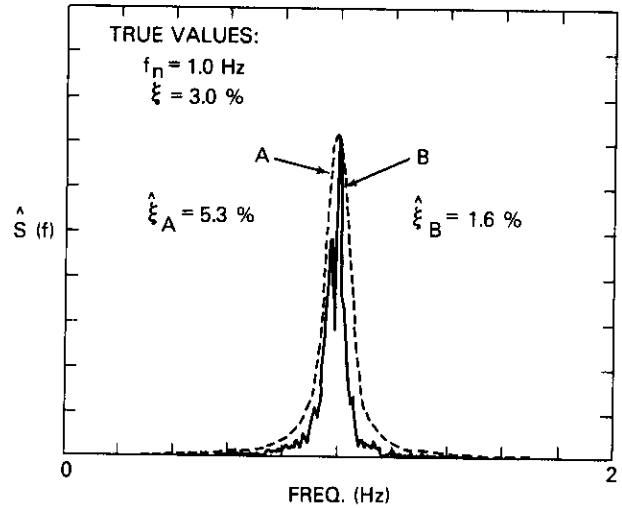

Fig. 1 - Conventional estimates of 1 dof system response spectrum.

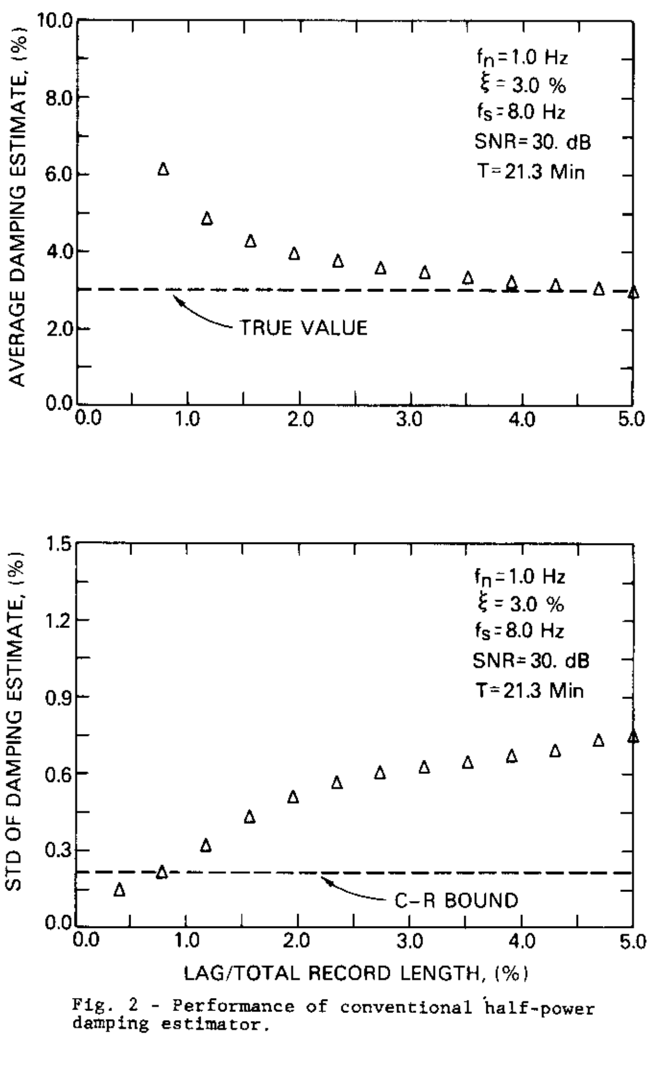

Fig. 2 - Performance of conventional half-power damping estimator.

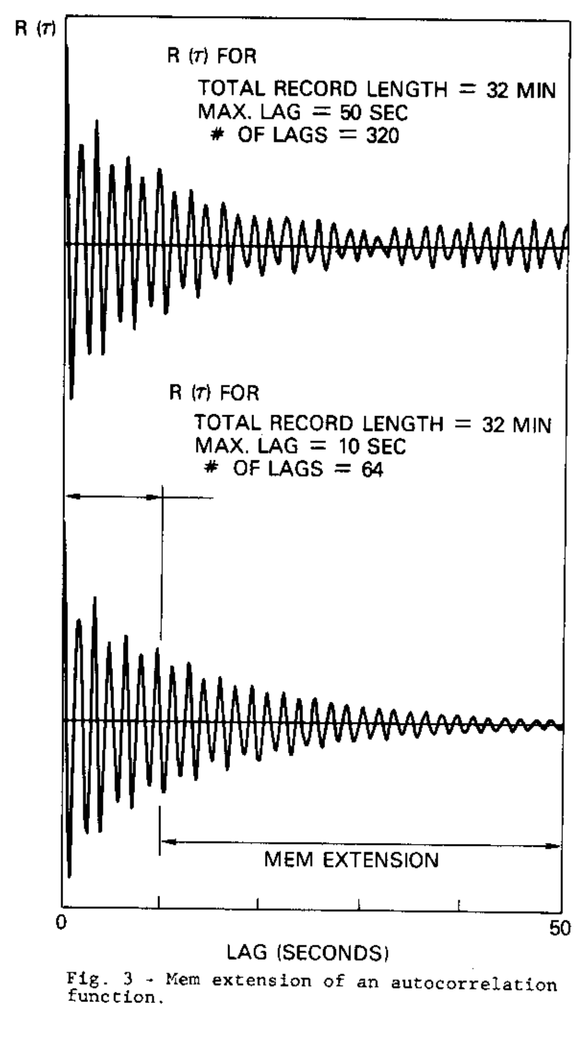

Fig. 3 - Mem extension of an autocorrelation function.

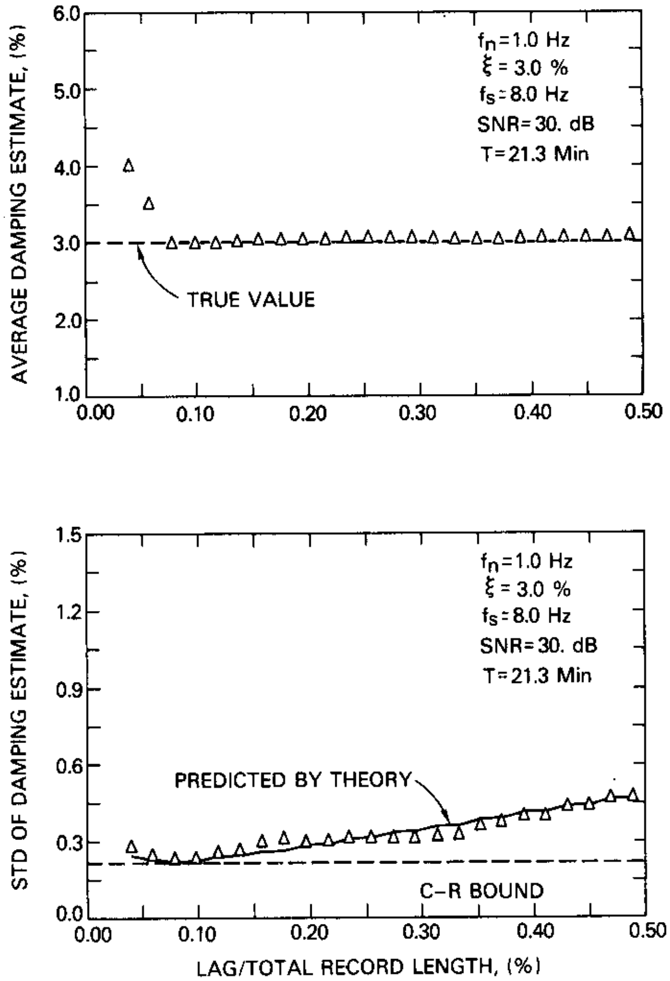

Fig. 4 - Performance of the mem damping estimator - 1 dof.

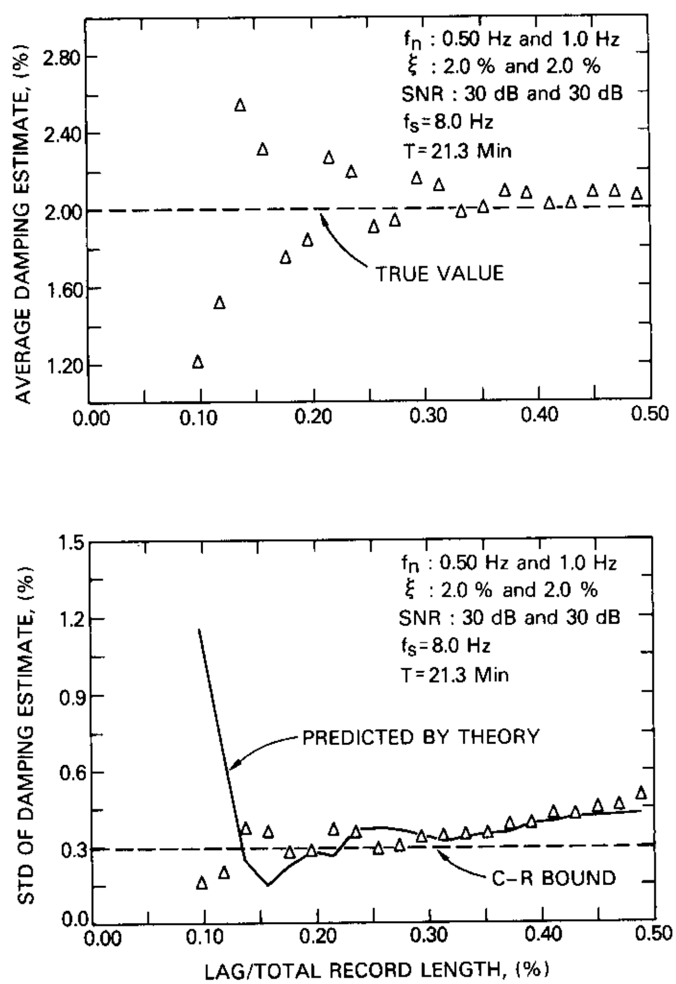

Fig. 5 - Performance of the mem damping estimator - 2 dof.

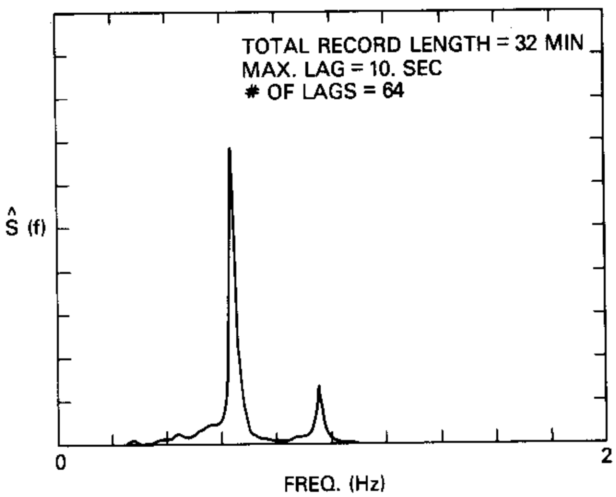

Fig. 6 - mem broadside response spectrum for south pass 62 c.

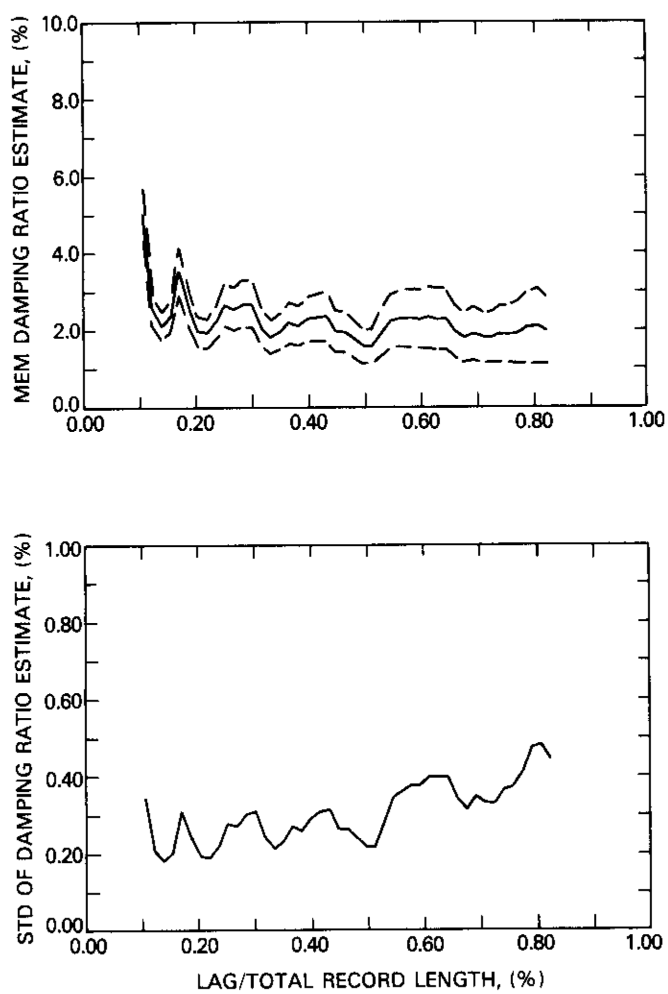

Fig. 7 - mem damping ratio estimates - 1st broadside flexure mode.